# Flow and heat transfer in the boundary layer of a micropolar fluid on a continuous moving surface

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**Abstract** *A boundary layer analysis is presented for the fluid flow and heat transfer characteristics of an incompressible micropolar fluid flowing over a plane moving surface in parallel or in reverse to the free stream. The isothermal boundary condition has been treated in this paper. The resulting system of non-linear ordinary differential equations is solved by the multi-stage continuous Runge-Kutta method with shooting techniques. Numerical results are obtained for the velocity, angular velocity and temperature distributions. The results indicate that micropolar fluids display drag reduction and heat transfer reduction characteristics.*

## Nomenclature

| | |
|---|---|
| $T_w, T_\infty$ | = wall temperature, free stream temperature |
| $Re_w$ | = Reynolds number, $\frac{u_w x}{v}$ |
| $Re_\infty$ | = Reynolds number, $\frac{u_\infty x}{v}$ |
| N | = angular velocity |
| f | = dimensionless stream function |
| g | = dimensionless angular velocity |
| (x, y) | = rectangular Cartesian coordinates |
| Pr | = Prandtl number, $\frac{v}{\alpha}$ |
| T | = fluid temperature |
| j | = micro inertia |
| $\kappa$ | = material coefficient |
| Nu | = Nusselt number |
| u | = velocity component in x-direction |
| v | = velocity component in y-direction |

*Greek symbols*

| | |
|---|---|
| $\rho$ | = density |
| $\alpha$ | = thermal diffusivity |
| $\psi$ | = stream function |
| $\Delta$ | = micropolar parameter, $\frac{\kappa}{\mu}$ |
| $\mu$ | = dynamic viscosity |
| $\nu$ | = kinematics viscosity |
| $\theta$ | = dimensionless temperature |
| $\eta$ | = similarity variable |
| B | = dimensionless parameter, $\frac{x^2}{(R_{ew}+R_{e\infty})j}$ |
| $\lambda$ | = dimensionless parameter, $\frac{2\gamma}{\mu j}$ |
| $\gamma$ | = viscosity coefficient of the fluid |
| $\xi$ | = dimensionless parameter, $\left(1+\frac{Re_\infty}{Re_w}\right)^{-1}$ |

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## Introduction

Boundary-layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering applications, for example, the aerodynamic extrusion of plastic sheets, the cooling of a metallic plate in a cooling bath, the boundary layer along material handling conveyers, and the boundary layer along a liquid film in condensation processes. In view of these applications, Sakiadis (1969) initiated the theoretical study of boundary layer on a continuous semi-infinite sheet moving steadily through an otherwise quiescent fluid environment. An experimental and theoretical treatment was made for the flow past a continuous flat surface by Tsou *et al.* (1967) who determined heat transfer rates for certain values of the Prandtl number. Choi (1982) considered the axisymmetric boundary layer flow generated by a moving, continuous, hot cylinder of infinite length issuing from a slot into stagnant air at uniform temperature. Karwe and Jaluria (1986) carried out a numerical and analytical study of the transport process arising due to the movement of a continuous heated body. Abdelhafez (1989) and Chappidi and Gunnerson (1989) studied the boundary layer flow on a continuous flat surface, where both the free stream and plate velocities are constant and are moving in the same direction. Afzal *et al.* (1993) considered the momentum and heat transfer on a continuous flat surface moving in a parallel stream by using a composite velocity of the free stream and wall velocities. Kumari and Nath (1996) investigated the momentum and thermal boundary layers over a semi-infinite flat plate when the external stream as well as the plate is impulsively moved with constant velocities. Lin and Hung (1994) studied the flow and heat transfer of a plane surface moving in parallel and in reverse to the free stream.

All the above investigations were restricted to Newtonian fluid. Due to the increasing importance in the processing industries and elsewhere of materials whose flow behavior in shear cannot be characterized by Newtonian relationships, a new stage in the evolution of fluid dynamic theory is in progress. Hoyt and Fabula (1984), and Vogel and Patterson (1964) conducted experiments with fluids containing minute amounts of polymeric additives. These experiments indicated that fluids with additives display a reduction in skin friction near a rigid body. The Newtonian fluid mechanics cannot explain this phenomenon. Therefore, Eringen (1966) has proposed the theory of micropolar fluids taking into account the inertial characteristics of the substructure particles, which are allowed to undergo rotation. This theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood, etc. Eringen (1972) extended the micropolar fluid theory and developed the theory of thermomicropolar fluids. The boundary layer concept in such fluids was first studied by Peddieson and McNitt (1970) and Wilson (1970). Mathur *et al.* (1978) studied steady thermal boundary-layer flow past a circular cylinder whose axis is placed normal to an oncoming free stream of an incompressible micropolar fluid. Gorla (1983) investigated the steady boundary layer flow of a micropolar fluid at a two-dimensional stagnation point on a moving wall. Hassanien and Gorla (1990) investigated the heat transfer characteristics of a steady, incompressible, micropolar fluid flowing past a non isothermal stretching sheet with suction and blowing. Gorla and Hassanien (1990) analyzed the boundary layer flow of a micropolar fluid in the vicinity of an axisymmetric stagnation flow on a moving cylinder.

The present work was undertaken in order to study the flow and heat transfer of the micropolar fluid on a plane surface moving in parallel and in reverse to the free stream. The development of the velocity, angular velocity and temperature distributions have been illustrated for several values of $\Delta$ and $\xi$.

**Analysis**

Consider a steady, laminar, incompressible, micropolar fluid flow over a plane surface, which is moving in a parallel or reverse to a free stream of a uniform velocity $u_\infty$. The plane surface is assumed to move with a uniform velocity of $u_w$ and is maintained at a constant temperature $T_w$. Assume the surface and the free stream are at the same temperature or with small temperature difference so that the buoyancy effect on flow is negligible. The physical properties of fluid are assumed to be constant.

Under the assumption of the boundary layer, the governing equations, as given by Eringen (1966; 1972) may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\nu + \frac{\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = -\kappa\left(2N + \frac{\partial u}{\partial y}\right) + \gamma\frac{\partial^2 N}{\partial y^2} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

With boundary conditions

$$u = \pm u_w, v = 0, T = T_w, N = 0 \quad at \quad y = 0$$

$$u \to u_\infty, T \to T_\infty, N \to 0 \quad as \quad y \to \infty$$

In the above equations, x and y are coordinates parallel and normal to the plate respectively; u, v are the velocity components in x and y directions respectively; N is the angular velocity; T is the temperature; $\nu, k$ and $\gamma$ are the kinematic, rotational and gyro viscosity coefficients respectively; j is the microinertia per unit mass and $\alpha$ is the thermal diffusivity of the fluid.

A comment will be made on the boundary condition of microrotation at the wall as given by the equation: N (x, 0) = n $\frac{\partial u}{\partial y}$. When n = 0, we obtain N(x, 0) = 0. This represents the case of concentrate particle flows in which the microelements close to the wall are not able to rotate. The case corresponding to n=1/2 results in the vanishing of the antisymmetric part of the stress tensor and represents weak concentrations. The particle spin is equal to fluid vorticity at the boundary for fine particle suspensions. The case corresponding to n = 1 is representative of turbulent boundary layer flows. Thus, for n = 0 particles are not free to rotate near the surface, whereas, as n increases to 0.5 and 1, the microrotation term gets augmented and induces flow enhancement. In this paper, we have considered the case corresponding to n = 0 only.

Proceeding with the analysis, we define the following transformations:

$$\eta = \left(\frac{y}{x}\right)(R_{e_w} + R_{e_\infty})^{\frac{1}{2}}, f = \frac{\psi}{\nu(R_{e_w} + R_{e_\infty})^{\frac{1}{2}}}$$

$$\xi = \frac{u_w}{(u_w + u_\infty)} = \left(1 + \frac{u_\infty}{u_w}\right)^{-1} = \left(1 + \frac{R_{e\infty}}{R_{e_w}}\right)^{-1}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$N = \frac{\nu}{x^2}(R_{e_w} + R_{e_\infty})^{\frac{3}{2}} g(\eta)$$

Equations (2), (3) and (4) are transformed in terms of the variable $\eta$ as:

$$2(1+\Delta)f''' + ff'' + 2\Delta g(\eta) = 0 \tag{5}$$

$$\lambda g'' - 2\Delta B(2g + f'') + f'g + g'f = 0 \tag{6}$$

$$2\theta'' + Pr\,\theta' f = 0 \tag{7}$$

The transform boundary conditions are given as:

$$f(0) = 0, \; f'(0) = \pm\xi, \; f'(\infty) = 1 - \xi \tag{8}$$

$$g(0) = 0, \; g(\infty) = 0, \; \theta(0) = 1, \; \theta(\infty) = 0 \tag{9}$$

In the above equations, a prime denotes differentiating with respect to $\eta$.The equations (5-7) together with the boundary conditions (8) and (9) are solved numerically by using the multi-stage continuous implicit Runge-Kutta method with Shooting method.

**Method of solution**

Initial value problems arise in many applications in engineering fluid mechanics. Several schemes are proposed to solve such problems. For a survey of methods, we refer to the work of Alexander (1977) and El-Gendi (1969). In this paper, we address the question of numerical solution of stiff system of ordinary differential equations (ODEs). We apply the diagonally implicit Runge-Kutta methods (DIRK) for stiff system of ODEs.

El-Gendi (1969) used a finite Chebyshev expansion

$$y(t) = \sum_{r=0}^{q} a_r T_r(t) \tag{10}$$

where t $\varepsilon$ [-1,1] to evaluate the integral $\int_{-1}^{t} y(s)ds$ at points $t_I = -\cos(i\pi/q)$ where i = O (1) q.

The partition interval [0,$t_f$] is to be defined as $\Delta = \{0 = t_0 \prec t_1 \prec \ldots t_N = t_f\}$ with step size $\tau_i = t_{i+1} - t_i$. Each interval $[t_i, t_{i+1}]$ is divided by the Chebyshev collocation points into:

$$t_{ij} = t_i + h_i c_j \text{ , } c_j = \frac{1}{2}\left(1 - \cos\left(\frac{j\pi}{\upsilon}\right)\right) \text{ and } j = 0(1)\upsilon \tag{11}$$

The main aim of this method is to find approximate for

$$\int_{t_i}^{t} \underline{f}(s)ds \quad \text{and} \int_{t_i}^{t} \underset{k-\text{times}}{\cdots\cdots\cdots} \int_{t_i}^{t} \underline{f}(s)ds \quad \text{on each } I_i$$

Consider the approximate $\tilde{f}(t)$ for $f(t)$ as:

$$\tilde{f} = \sum_{r=0}^{\upsilon}{}^{//} \underline{a_r} T_r\left(\frac{2(t - t_i)}{h_i}\right) - 1 \tag{12}$$

Where

$$\underline{a_r} = \frac{2}{\upsilon} \sum_{l=0}^{\upsilon}{}^{//} \underline{\tilde{f}}\left(t_i + \frac{h_i}{2}\left(1 + \cos\left(\frac{l\pi}{\upsilon}\right)\right)\right) T_r\left(\cos\left(\frac{l\pi}{\upsilon}\right)\right) \tag{13}$$

such that

$$r = 0(1)\upsilon, t = t_i + \theta h_i, i = 0(1)N - 1 \quad \text{and} T_r(\theta) = \cos(r \cos^{-1}(\theta))$$

is the Chebyshev polynomial. A summation symbol with double primes denotes a sum with the first and last terms halved.

By using (12) and (13) and the following relation:

$$\int_0^{\theta} T_r(2s - 1)\, ds = \frac{1}{2}\begin{cases} \dfrac{T_{r+1}(2\theta - 1)}{2(r+1)} - \dfrac{T_{r-1}(2\theta - 1)}{2(r-1)} + \dfrac{(-1)^{r+1}}{r^2 - 1}, & r \geq 2 \\ \dfrac{1}{4}\{T_2(2\theta - 1) - 1\}, & r = 1 \\ T_1(2\theta - 1) + 1, & r = 0 \end{cases}$$

We obtain

$$\int_{t_i}^{t_i+\theta h_i} \tilde{\underline{f}}(s)\, ds = h_i \sum_{l=0}^{\upsilon} b_l(\theta)\, \tilde{\underline{f}}(t_{il}) \tag{14}$$

Where

$$\begin{aligned} b_l(\theta) = & \frac{1}{2\,\upsilon(1+\delta_{\upsilon l}+\delta_{0l})} \left\{ \sum_{j=0}^{\upsilon} \frac{T_j(x_l)}{(1+\delta_{\upsilon j})(j+1)} \left( T_{j+1}(2\,\theta-1) + (-1)^j \right) \right. \\ & \left. - \sum_{j=2}^{\upsilon} \frac{T_j(x_l)}{(1+\delta_{\upsilon j})(j-1)} \left( T_{j-1}(2\,\theta-1) + (-1)^j \right) \right\}, l = 0(1)\upsilon \end{aligned} \tag{15}$$

$x_l = -\cos\left(\frac{l\pi}{\upsilon}\right)$ and $\delta_{lk}$ is the kronecker delta.

By differentiating $b_1(\theta)$ with respect to $\theta$ we obtain

$$b_l'(\theta) = \frac{2}{\upsilon(1+\delta_{\upsilon l}+\delta_{0l})} \sum_{j=0}^{\upsilon}{}^{//}\, T_j\left(-\cos\left(\frac{l\pi}{\upsilon}\right)\right) T_j(2\,\theta-1) =$$

$$\begin{cases} \delta_{jl} & \theta = c_j \\ (-1)^{\upsilon-1} \left\{ \dfrac{T_{\upsilon+1}(2\,\theta-1) - T_{\upsilon-1}(2\,\theta-1)}{4\upsilon(1+\delta_{\upsilon l}+\delta_{0l})(\theta-c_l)} \right\} & \theta \neq c_l\, \forall l = 0(1)\upsilon \end{cases} \tag{16}$$

From (11), (14), (15) and (16)

$$\int_{t_i}^{t_{ij}} \underline{f}(s)\, ds = h_i B\, \underline{F}_i\,,$$

$$\int_{t_i}^{t_{ij}} \cdot\cdot \underset{m-\text{times}}{\cdots\cdots} \cdot\cdot \int_{t_i}^{t_{ij}} \underline{f}(s)\, ds = \frac{(h_i)^m}{(m-1)!} Bk\underline{F}_i \text{ and}$$

$$\int_{t_0}^{t_{ij}} \underline{f}(t,s)\, ds = \sum_{l=0}^{i-1} h_l \sum_{s=0}^{\upsilon} b_{\upsilon s}\, \underline{f}(t, t_{ls}) + h_i \sum_{s=0}^{\upsilon} b_{j\,s}\, \underline{f}(t, t_{is}) \qquad j = 0(1)\upsilon$$

Where

$$k = \text{diag}\left[ \left(c_j - c_0\right)^{m-1}, \ldots\ldots\ldots, \left(c_j - c_\upsilon\right)^{m-1} \right],$$

$$B = \left[b_{js}\right]_{j,s=0}^{\upsilon}, b_{j\,s} = b_s\left(c_j\right) \text{ and } \underline{F}_i = \left[\underline{f}(t_{i0}), \ldots\ldots\ldots\ldots, \underline{f}(t_{i\upsilon})\right]^T$$

We may now write

$$\underline{y}(t) = \underline{y}(t_i) + \int_{t_i}^{t} \underline{z}(s)\,ds, \qquad \underline{z}(t) = \underline{f}\Big(t, \underline{y}(t)\Big)$$

suppose that $\tilde{y}(t)$ is the approximation of $\underline{y}(t)$ and

$$\tilde{z}(t_{il}) = f(t_{il}, \tilde{y}(t_{il})) \qquad l = 1(1)\upsilon$$

on $[t_i, t_{i+1}]$ where

$$\tilde{y}(t) = \begin{cases} g(t_0) & t = t_0 \\ \tilde{y}\big(t_{j-1,\upsilon}\big) + h_j b_0(\theta)\,\tilde{z}\big(t_{j-1,\upsilon}\big) + h_j \sum_{l=1}^{\upsilon} b_l(\theta)\tilde{z}\big(t_{ji}\big) \end{cases}$$

Where $t = t_j + \theta h_j \in I_j, b_j(0) = 0, j = 0(1)\upsilon$ and $\sum_{s=0}^{\upsilon} b_s(\theta) = \theta$

The method is similar to the implicit Runge-Kutta method of numerical integration.

## Discussion

The numerical results for the velocity and temperature fields are obtained for $\lambda = 1.5$, B = 0.01, pr = 0.7, $\Delta$ ranging from 0 to 4.5 and $\xi$ ranging from 0 to 1.

The wall shear and couple stresses are written as

$$\tau_w = (\mu + \kappa)\frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$\tau_w = \rho\left(\frac{\nu}{x}\right)^2 (Re_w + Re_\infty)^{\frac{3}{2}}(1+\Delta)\,f''(o)$$

$$m_w = \gamma\frac{\partial N}{\partial y}\bigg|_{y=0}$$

$$m_w = \gamma\left(\frac{\nu}{x^3}\right)(Re_w + Re_\infty)^2\,g'(0)$$

The Nusselt number can be written as

$$Nu = -\frac{x}{(T_w - T_\infty)}\frac{\partial T}{\partial y}\bigg|_{y=0}$$

$$Nu = -(Re_w + Re_\infty)^{\frac{1}{2}}\,\theta'(0)$$

Figures 1-3 represent the distributions of velocity within the boundary layer in the case of the moving surface in parallel to the free stream. As the material

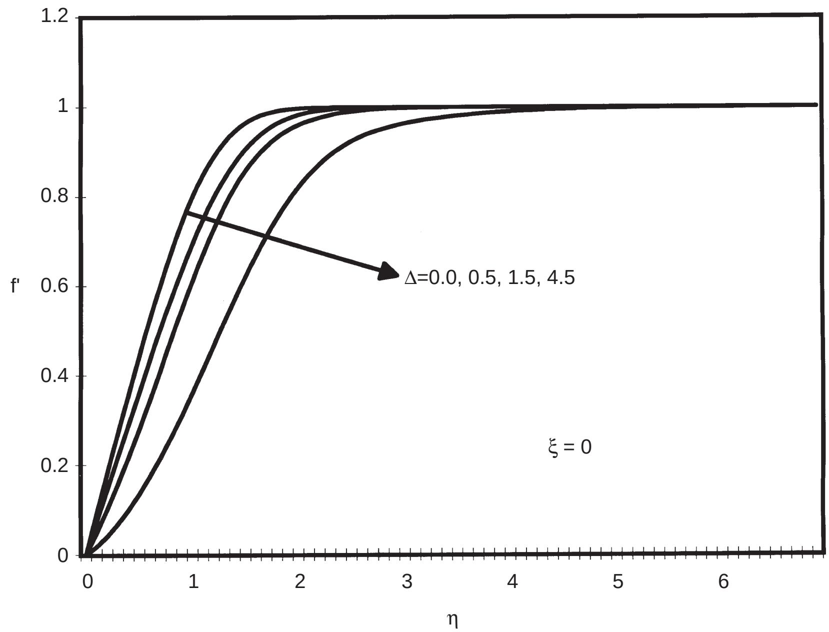


**Figure 1.**
Velocity distribution for $\xi = 0.0$ (parallel state)

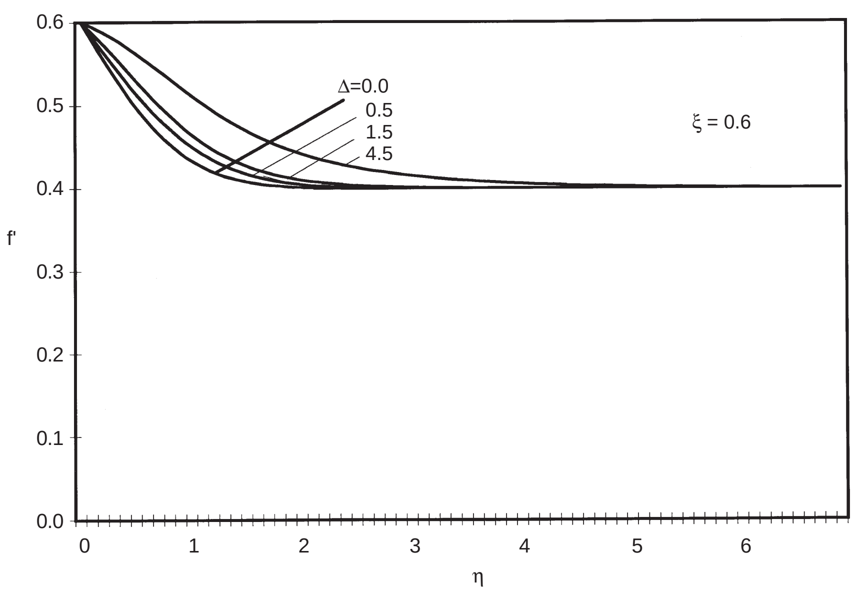


**Figure 2.**
Velocity distribution for $\xi = 0.6$ (parallel state)

parameter $\Delta$ increases we observe that the velocity distribution becomes less uniform within the boundary layer. The thickness of boundary layer increases as the material parameter $\Delta$ increases.

Figures 4-6 display the effect of continuous, moving plate in parallel to free stream on the microrotation component. As the material parameter $\Delta$

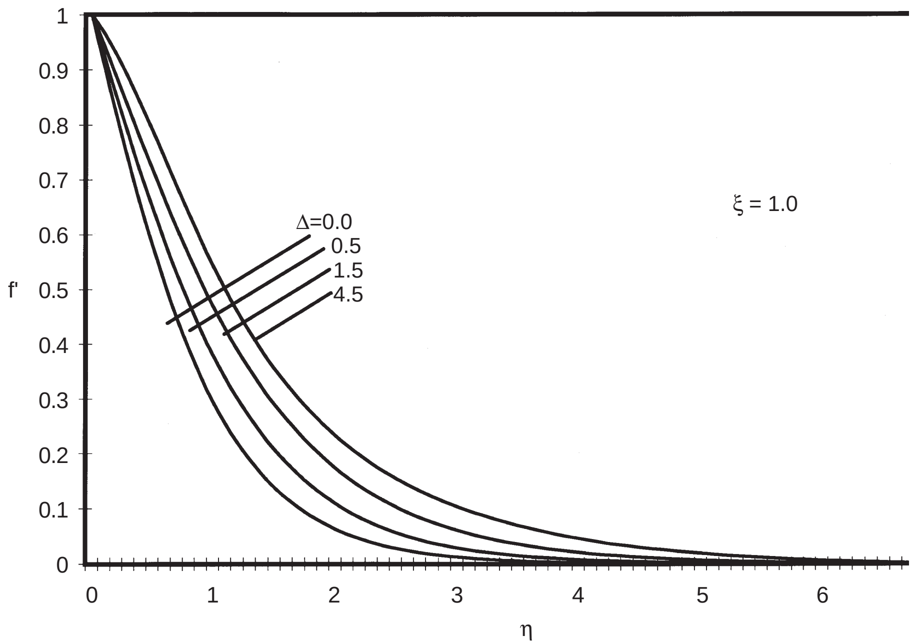


**Figure 3.**
Velocity distribution for $\xi = 1.0$ (parallel state)

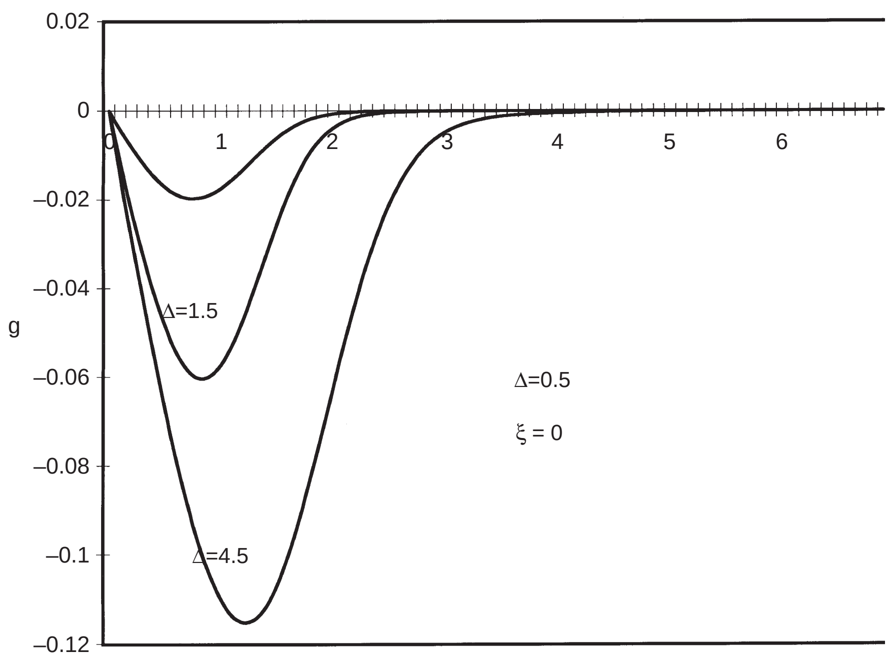


**Figure 4.**
Angular velocity distribution for $\xi = 0.0$ (parallel state)

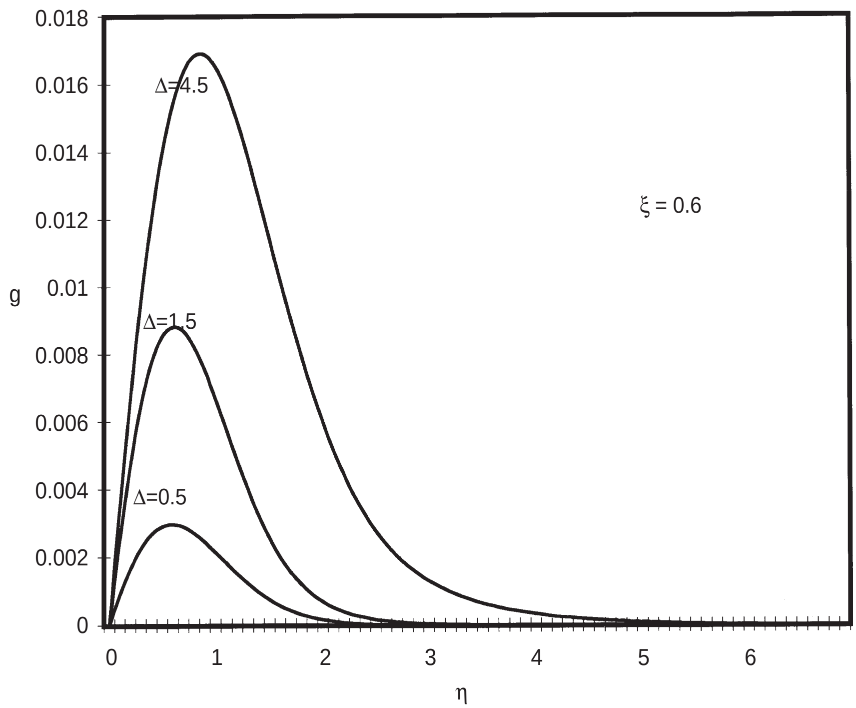


**Figure 5.**
Angular velocity distribution for $\xi = 0.6$ (parallel state)

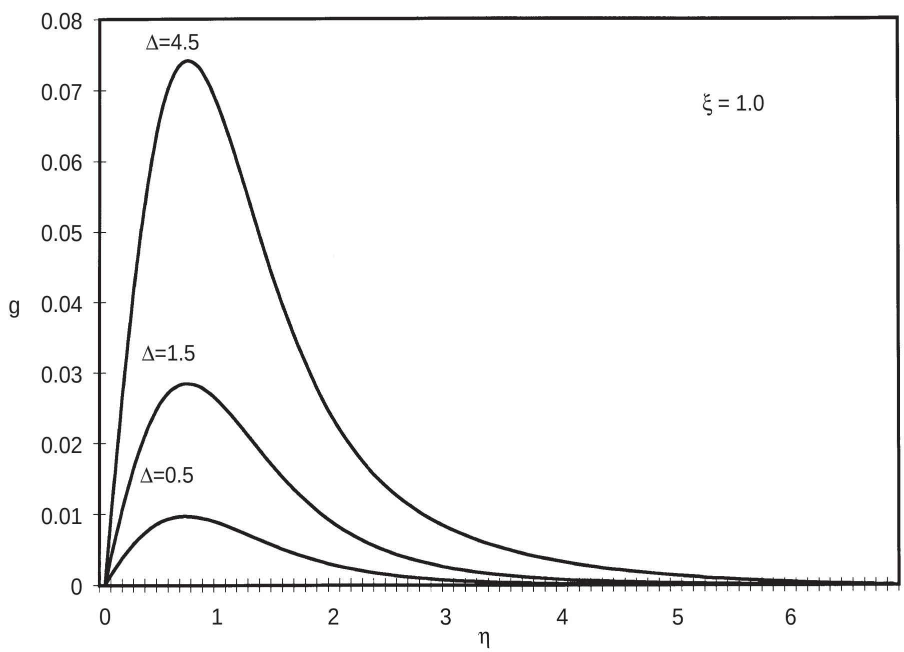


**Figure 6.**
Angular velocity distribution for $\xi = 1.0$ (parallel state)

increases the magnitude of angular velocity also increases. Figures 7-9 describe the temperature distributions within the thermal boundary layer for moving surface in parallel to free stream. The temperature distributions become more uniform within the thermal boundary layer as the material parameter $\Delta$ decreases.

Figure 10 shows the velocity distribution for moving surface in reverse to free stream. The velocity distribution becomes more uniform within the boundary layer as the material parameter $\Delta$ decreases. The thickness of boundary layer increases with the material parameter $\Delta$. Figure 11 indicates that the magnitude of angular velocity decreases as the material parameter $\Delta$ increases for moving surface in reverse to free stream.

Figure 12 displays the temperature distribution for the moving surface in reverse direction to the free stream. The temperature takes more uniform shape with increasing value of the material parameter $\Delta$. As the material parameter $\Delta$ increases, the thickness of the thermal boundary layer increases.

The missing values of the velocity, angular velocity and thermal functions for the parallel state are contained in Table I but for the reverse state in Table II. The results in Table I indicate that the friction factor and the heat transfer rate decrease as the material parameter $\Delta$ increases. The results show that the wall couple stress decreases with increasing values of the material parameter $\Delta$. The results for the reverse flow in Table II show that the friction factor is not sensitive to $\Delta$. As the

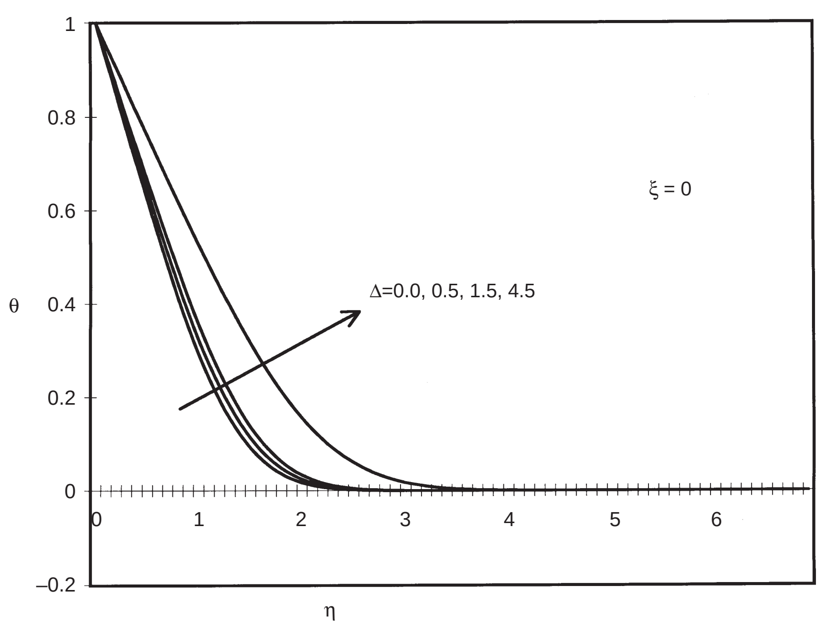


**Figure 7.**
Temperature distribution for $\xi = 0.0$ (parallel state)

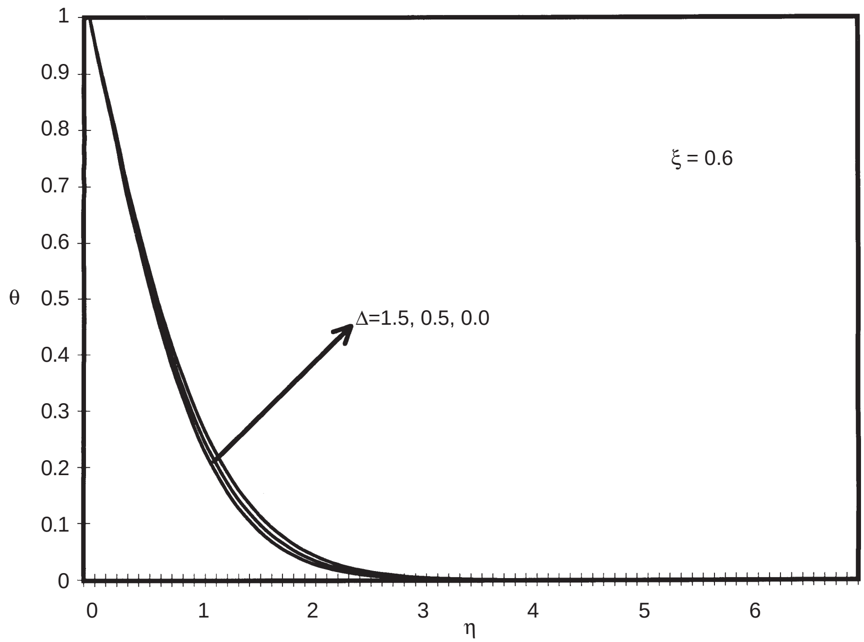


**Figure 8.**
Temperature distribution for $\xi = 0.6$ (parallel state)

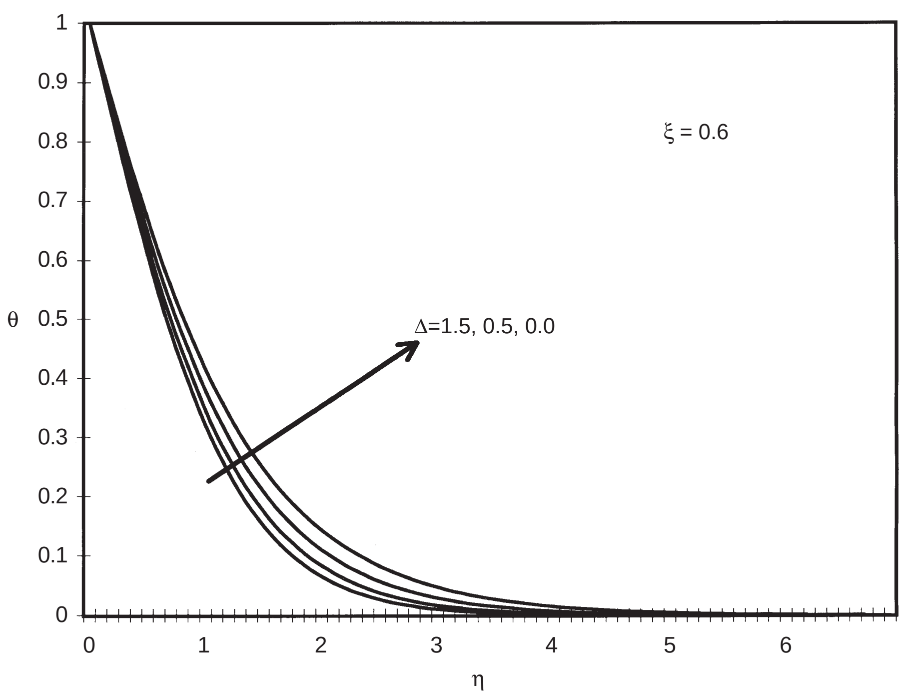


**Figure 9.**
Temperature distribution for $\xi = 1.0$ (parallel state)

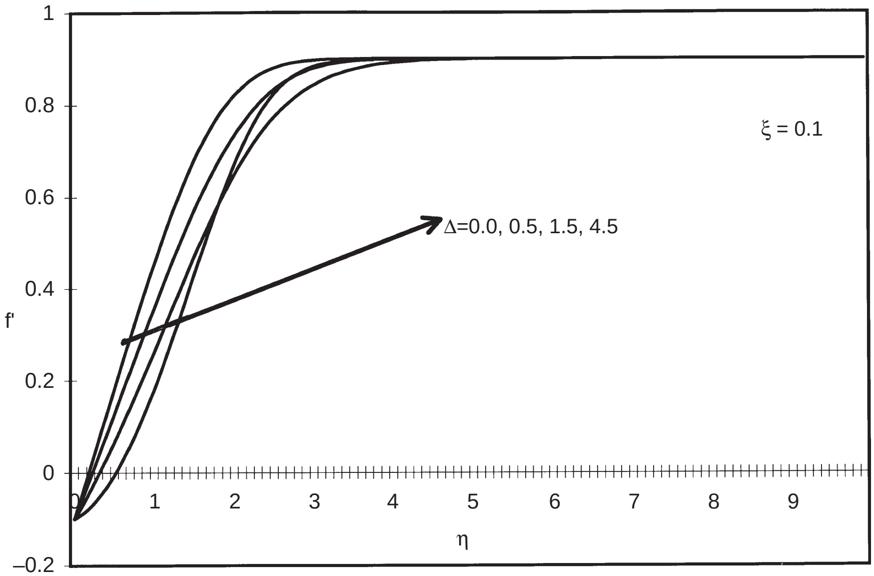


**Figure 10.**
Velocity distribution for $\xi = 0.1$ (reverse state)

material parameter $\Delta$ increases the wall couple stress decreases whereas the heat transfer rate increases at $\xi = 0.0$ but decreases at $\xi = 0.05$, 0.1. We may note that $\Delta = 0$ case corresponds to Newtonian fluids. By comparing the Newtonian fluid data with micropolar fluid data, from these tables, we observe that micropolar fluids display reduced drag and heat transfer rate.

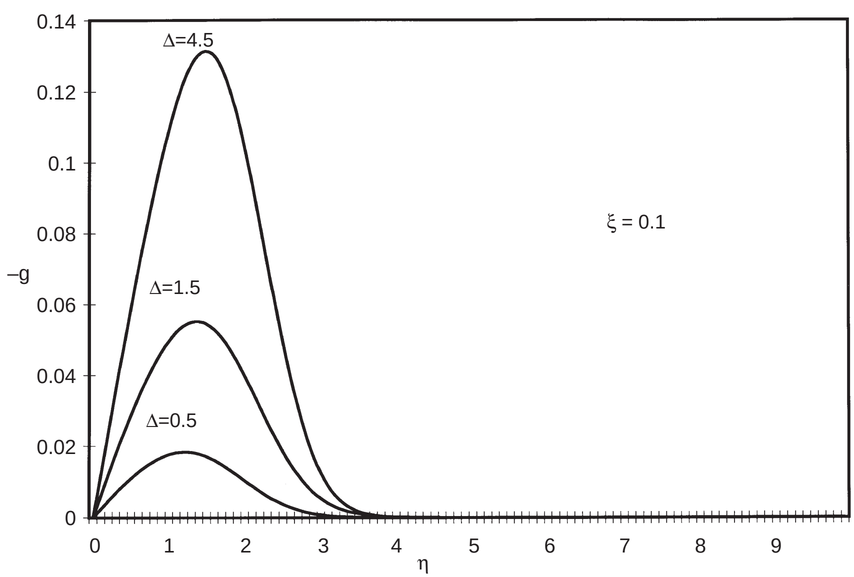


**Figure 11.**
Angular velocity distribution for $\xi = 0.1$ (reverse state)

| $\xi$ | $\Delta$ | $f''(0)$ | $g'(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|
| 0.0 | 0.0 | 0.9001 | 0.0 | 0.1088 |
| | 0.5 | 0.7086 | –0.0441 | 0.1026 |
| | 1.5 | 0.4484 | –0.1173 | 0.0961 |
| | 4.5 | 0.15496 | –0.1455 | 0.1199 |
| 0.1 | 0.0 | 0.7542 | 0.0 | 0.1124 |
| | 0.5 | 0.5956 | –0.0358 | 0.1074 |
| | 1.5 | 0.3872 | –0.0963 | 0.1023 |
| | 4.5 | 0.1419 | –0.1226 | 0.1299 |
| 0.2 | 0.0 | 0.5885 | 0.0 | 0.1155 |
| | 0.5 | 0.4662 | –0.0271 | 0.1117 |
| | 1.5 | 0.30903 | –0.0737 | 0.1079 |
| | 4.5 | 0.1177 | –0.0956 | 0.1393 |
| 0.3 | 0.0 | 0.4064 | 0.0 | 0.1182 |
| | 0.5 | 0.3227 | 0.0183 | 0.1157 |
| | 1.5 | 0.2171 | –0.05003 | 0.1131 |
| | 4.5 | 0.0851 | –0.0658 | 0.1481 |
| 0.4 | 0.0 | 0.2097 | 0.0 | 0.1206 |
| | 0.5 | 0.1669 | –0.0092 | 0.1193 |
| | 1.5 | 0.1136 | –0.0025 | 0.11802 |
| | 4.5 | 0.00454 | –0.00337 | 0.1566 |
| 0.5 | 0.0 | 0.0 | 0.0 | 0.1647136 |
| | 0.5 | 0.0 | 0.0 | 0.1647136 |
| | 1.5 | 0.0 | 0.0 | 0.1647136 |
| | 4.5 | 0.0 | 0.0 | 0.1647136 |
| 0.6 | 0.0 | –0.2213 | 0.0 | 0.1243 |
| | 0.5 | –0.1766 | 0.0093 | 0.1257 |
| | 1.5 | –0.1231 | 0.02601 | 0.12707 |
| | 4.5 | –0.5046 | 0.0351 | 0.1727 |
| 0.7 | 0.0 | –0.45307 | 0.0 | 0.1257 |
| | 0.5 | –0.3621 | 0.0188 | 0.1286 |
| | 1.5 | –0.2523 | 0.0525 | 0.1313 |
| | 4.5 | –0.1052 | 0.0714 | 0.1806 |
| 0.8 | 0.0 | –0.69404 | 0.0 | 0.1267 |
| | 0.5 | –0.5553 | 0.0285 | 0.1312 |
| | 1.5 | –0.38907 | 0.0795 | 0.1353 |
| | 4.5 | –0.1639 | 0.1086 | 0.1883 |
| 0.9 | 0.0 | –0.9430 | 0.0 | 0.1271 |
| | 0.5 | –0.5699 | 0.0211 | 0.1803 |
| | 1.5 | –0.4209 | 0.0593 | 0.1888 |
| | 4.5 | –0.2260 | 0.1465 | 0.1960 |
| 1.0 | 0.0 | –0.8957 | 0.0 | 0.1715 |
| | 0.5 | –0.7247 | 0.0265 | 0.1834 |
| | 1.5 | –0.5366 | 0.0174 | 0.1944 |
| | 4.5 | –0.2911 | 0.1849 | 0.2035 |

**Table I.**
Values of $f''(0), g'(0)$ and $-\theta'(0)$ for various values of $\Delta$ and $\xi$ for parallel moving surface

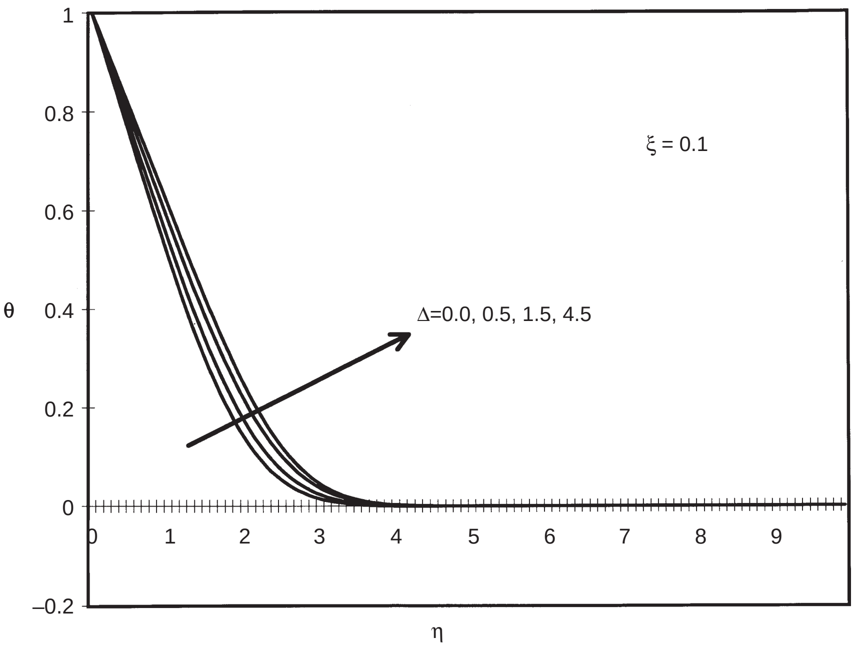


**Figure 12.** Temperature distribution for $\xi = 0.1$ (reverse state)

## Concluding remarks

The theory of micropolar fluids due to Eringen is used to formulate a set of equations for the micropolar fluid flow and heat transfer in a parallel or reverse to a free stream. We have considered isothermal boundary condition in this paper. The governing boundary layer equations are solved numerically. The development of the temperature, the velocity and angular velocity distributions has been illustrated. A discussion is provided for the effect of the material

| $\xi$ | $\Delta$ | $f''(0)$ | $-g'(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|
| 0.0 | 0.0 | 1.0 | 0.0 | 0.1087 |
| | 0.5 | 1.0 | 0.0252 | 0.1369 |
| | 1.5 | 0.9999 | 0.0673 | 0.1278 |
| | 4.5 | 1.0 | 0.1455 | 0.1986 |
| 0.05 | 0.0 | 0.95 | 0.0 | 0.1011 |
| | 0.5 | 0.95 | 0.02501 | 0.1262 |
| | 1.5 | 0.9499 | 0.0653 | 0.1166 |
| | 4.5 | 0.9499 | 0.1355 | 0.1085 |
| 0.1 | 0.0 | 0.8999 | 0.0 | 0.1234 |
| | 0.5 | 0.8999 | 0.0246 | 0.1143 |
| | 1.5 | 0.9 | 0.0626 | 0.1039 |
| | 4.5 | 0.8999 | 0.12207 | 0.0954 |

**Table II.** Values of $f''(0), g'(0)$ and $\theta'(0)$ for various values of $\Delta$ and $\xi$ for reverse moving surface parameter $\Delta$ on the boundary layer and missing values of velocity, angular velocity and thermal functions are tabulated for a wide range of the material parameter $\Delta$ and dimensionless parameter $\xi$. The results indicate that micropolar fluids display drag reduction and heat transfer reduction characteristics.

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